Consider the polar equation $r = 2 + 2\cos 3\theta$.

SCORE: _____ / 30 PTS

The following symmetry tests do NOT indicate that the graph is symmetric: $(-r, \theta)$, $(-r, -\theta)$ and $(r, \pi+\theta)$

POLE $\theta = \frac{\pi}{2}$ POLE

[a] **Using the results above, along with the tests and shortcuts shown in lecture**, determine if the graph is symmetric over the polar axis, $\theta = \frac{\pi}{2}$ and/or the pole. Summarize your conclusions in the table on the right.

NOTE: Run as FEW tests as needed to prove your conclusions are correct.

$\theta = \frac{\pi}{2}$: $(r, \pi-\theta)$ $r = 2 + 2\cos 3(\pi-\theta)$

$r = 2 + 2\cos(3\pi - 3\theta)$

$r = 2 + 2[\cos 3\pi \cos 3\theta + \sin 3\pi \sin 3\theta]$ ($\cos 3\pi = -1$, $\sin 3\pi = 0$)

$r = 2 - 2\cos 3\theta$

POLAR AXIS : $(r, -\theta)$ $r = 2 + 2\cos 3(-\theta)$

$r = 2 + 2\cos 3\theta$

Type of symmetry	Conclusion
Over the polar axis	SYM
Over $\theta = \frac{\pi}{2}$	NO CONCL
Over the pole	NO CONCL

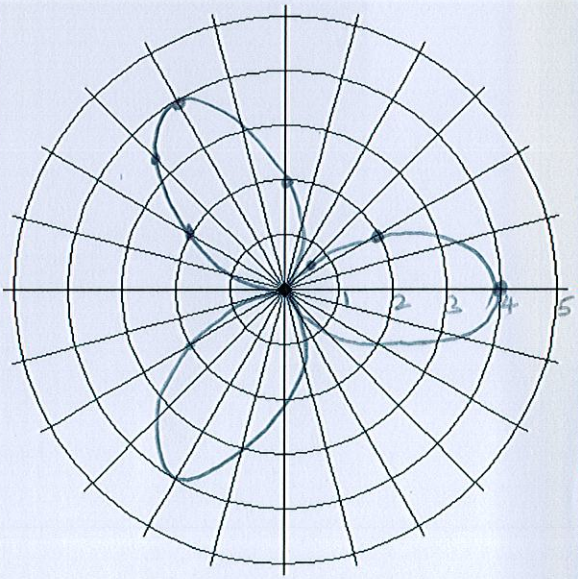
[b] Based on the results of part [a], what is the minimum interval of the graph you need to plot (before using reflections to draw the rest of the graph) ?

$[0, \pi]$

[c] The table below shows the value of r for all values of θ in the first quadrant. Find additional values of r if necessary and sketch the graph of the equation. **Label the scale on your axes clearly.**

θ	r
0	4
$\frac{\pi}{6}$	2
$\frac{\pi}{4}$	≈ 0.6
$\frac{\pi}{3}$	0
$\frac{\pi}{2}$	2

θ	r
π	0
$\frac{5\pi}{6}$	2
$\frac{3\pi}{4}$	≈ 3.4
$\frac{2\pi}{3}$	4



Convert the polar equation $r = 1 - \cot\theta$ to rectangular and simplify.

SCORE: _____ / 15 PTS

$$\sqrt{x^2+y^2} = 1 - \frac{x}{y}$$

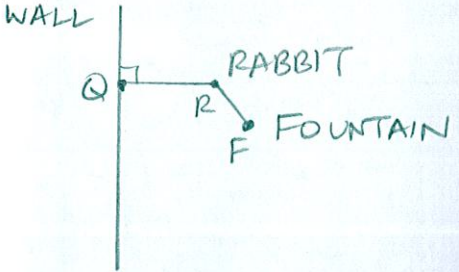
$$y\sqrt{x^2+y^2} = y - x$$

$$y^2(x^2+y^2) = (y-x)^2$$

A drinking fountain is 6 feet from the wall of a school building. A rabbit is running on the school grounds, so that it is always twice as far from the wall as it is from the fountain. What is the shape of the rabbit's path ?

SCORE: _____ / 10 PTS

Draw a diagram and write algebraic equations involving distances to justify your answer.



$RQ = 2RF$

$\frac{1}{2} = \frac{RF}{RQ} = e$

$\frac{1}{2} < 1 \rightarrow$ ELLIPSE

Consider the conic with rectangular equation $2x^2+8x-9y^2+54y-1=0$. **SCORE: ____ / 20 PTS**

[a] What is the shape of the graph of the equation ?

HYPERBOLA

[b] Find the co-ordinates of the focus/foci.

$2(x^2+4x+4)-9(y^2-6y+9)=1+8-81$

$2(x+2)^2-9(y-3)^2=-72$

$\frac{(y-3)^2}{8}-\frac{(x+2)^2}{36}=1$

FOCI

$(-2, 3\pm 2\sqrt{11})$

$c^2=8+36=44$

$c=2\sqrt{11}$

Consider the polar equation $r = \dfrac{60}{7+3\cos\theta}$. $= \dfrac{\frac{60}{7}}{1+\frac{3}{7}\cos\theta}$

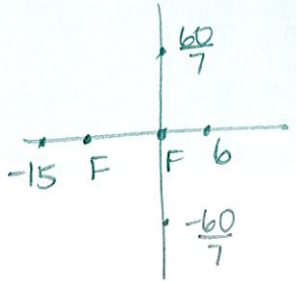
SCORE: _____ / 20 PTS

[a] What is the shape of the graph of the equation ?

$e = \frac{3}{7} < 1 \longrightarrow$ ELLIPSE

[b] Find the **rectangular** coordinates of the endpoints of **all** latera recta. **Do NOT convert the equation to rectangular.**

θ	r	
0	6	V
$\frac{\pi}{2}$	$\frac{60}{7}$	LR
π	15	LR
$\frac{3\pi}{2}$	$\frac{60}{7}$	V



OTHER FOCUS $(-15+6, 0) = (-9, 0)$

ENDPOINTS OF LR

$(0, \pm\frac{60}{7})$ $(-9, \pm\frac{60}{7})$

Consider the polar equation $r = 3 - 2\cos\theta$. $1 < \left|\frac{3}{2}\right| < 2$

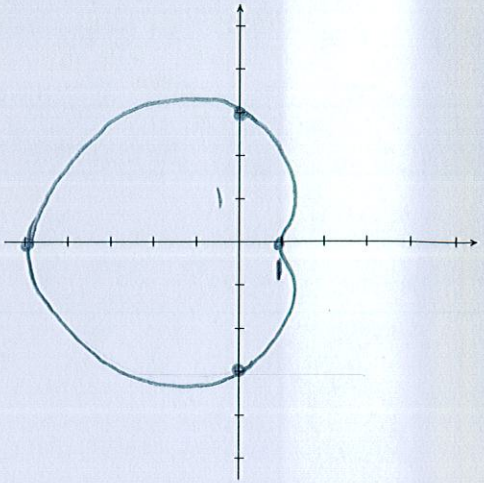
SCORE: ____ / 20 PTS

[a] What is the shape of the graph of the equation ?

LIMACON WITH DIMPLE

[b] Sketch the graph using the shortcut process shown in lecture / PowerPoint.
Label the scale on your axes clearly.

θ	r
0	1
$\frac{\pi}{2}$	3
π	5
$\frac{3\pi}{2}$	3



Find the **rectangular** equation of the ellipse with foci $(-5, 7)$ and $(-5, -1)$ and minor axis of length 8. **SCORE: _____ / 15 PTS**

CENTER $\left(-5, \frac{7+-1}{2}\right) = (-5, 3)$

$$\frac{(x+5)^2}{16} + \frac{(y-3)^2}{32} = 1$$

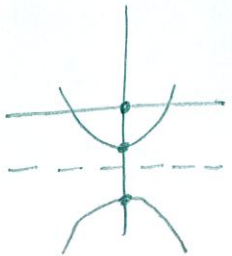
$4^2 + 4^2 = 32$

A hyperbola has a focus at the pole and vertices with **rectangular** co-ordinates $(0,-3)$ and $(0,-7)$. **SCORE: ____ / 20 PTS**

[a] Find polar co-ordinates for the vertices, using positive values of r and θ.

$(3, \frac{3\pi}{2})$ $(7, \frac{3\pi}{2})$

[b] Find the **polar** equation of the hyperbola. **Do NOT find the rectangular equation.**



$r = \frac{ep}{1-e\sin\theta}$

USE $(-7, \frac{\pi}{2})$ AND $(3, \frac{3\pi}{2})$

$-7 = \frac{ep}{1-e}$ $\quad 3 = \frac{ep}{1+e}$

$ep = -7+7e$ $\quad ep = 3+3e$

$-7+7e = 3+3e$

$e = \frac{5}{2}$ $\quad ep = 3+3(\frac{5}{2}) = \frac{21}{2}$

$r = \frac{\frac{21}{2}}{1-\frac{5}{2}\sin\theta}$

$r = \frac{21}{2-5\sin\theta}$